University of Chicago
Booth School of Business
41000: Business Statistics, Autumn 2021: Homework Assignment 1. Due in Week 3

Problem 1: Probability Answer the following statements TRUE or FALSE, providing a succinct explanation of your reasoning.

1. If the odds in favor of A are 3:5 then $P(A) = 0.4$.

2. You roll two fair three-sided dice.

   The probability the two dice show the same number is $1/4$.

3. If events A and B are independent and $P(A) > 0$ and $P(B) > 0$, then $P(A \text{ and } B) > 0$.

4. If $P(A \text{ and } B) \geq 0.5$ then $P(A) \leq 0.5$.

5. If two random variables have non-zero correlation, then they must be dependent.

6. If two random variables have zero correlation, then they must be independent.

7. If two random variables are independent, then the correlation between them must be zero.

Problem 2: Expectation and Strategy

An oil company wants to drill in a new location. A preliminary geological study suggests that there is a 20% chance of finding a small amount of oil, a 50% chance of a moderate amount and a 30% chance of a large amount of oil.

The company has a choice of either a standard drill that simply burrows deep into the earth or a more sophisticated drill that is capable of horizontal drilling and can therefore extract more but is far more expensive. The following table provides the payoff table in millions of dollars under different states of the world and drilling conditions

Oil	small	moderate	large
Standard Drilling	20	30	40
Horizontal Drilling	-20	40	80

Find the following

(a) The mean and variance of the payoffs for the two different strategies

(b) The strategy that maximizes their expected payoff

(c) Briefly discuss how the variance of the payoffs would affect your decision if you were risk averse

(d) How much are you willing to pay for a geological evaluation that would tell you with certainty the quantity of oil at the site prior to drilling?

Problem 3: Normal Distribution

After Facebook's earnings announcement we have the following distribution of returns. First, the stock beats earnings expectations 75% of the time, and the other 25% of the time earnings are in line or disappoint. Second, when the stock beats earnings, the probability distribution of percent changes is normal with a mean of 10% with a standard deviation of 5% and, when the stock misses earnings, a normal with a mean of -5% and a standard deviation of 8%, respectively.

(a) Ahead of the earnings announcement, what is the probability that Facebook stock will have a return greater than 5%?

(b) Do you get the same answer for the probability that it drops at least 5%?

Problem 4: Binomial Distribution

The Downhill Manufacturing company produces snowboards. The average life of their product is 10 years. A snowboard is considered defective if its life is less than 5 years. The distribution is approximately normal with a standard deviation for the life of a board of 3 years.

(a) What's the probability of a snowboard being defective?

(b) In a shipment of 120 snowboards, what is the probability that the number of defective boards is greater than 10?

[You can use `R` and simulation with `rbinom, rnorm` as an alternative]

2. **Problem 5: Portfolio ETF**

You want to build a portfolio of exchange traded funds (ETFs) for your retirement strategy. You?re thinking of whether to invest in growth or value stocks, or maybe a combination of both. Vanguard has two ETFs, one for growth (VUG) and one for value (VTV). The R script hwk1.R script on the course web-page let's you download historical price data.

- Plot the historical price series for VUG vs VTV.

- Calculate the means and standard deviations of both ETFs.

- Calculate their covariance.

- Suppose you decide on a portfolio that is a 50 / 50 split. Calculate the new mean and variance of your portfolio.

- Which portfolio best suits you?

- What's the probability that growth (VUG) will beat value (VTV) in the future?

Hint: you will find the following formulas useful. Let P denote the return on your portfolio which is a weighted combination $P = aX + bY$. Then

$$E(P) = aE(X) + bE(Y) \tag{1}$$

$$Var(P) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X, Y) \tag{2}$$

where $Cov(X, Y)$ is the covariance for X and Y .

Problem 6: Google

Visitors to your website are asked to answer a single survey http://www.google.com question before they get access to the content on the page. Among all of the users, there are two categories

1. Random Clicker (RC)

2. Truthful Clicker (TC)

There are two possible answers to the survey: yes and no.

Random clickers would click either one with equal probability. You are also giving the information that the expected fraction of random clickers is 0.3.

After a trial period, you get the following survey results. 65% said Yes and 35% said No.

(a) How many people people who are truthful clickers answered yes?