Name: OUTLINE SOLUTIONS

University of Chicago
Graduate School of Business
Business 410: Business Statistics

Special Notes:

1. This is a closed-book exam. You may use an 8×11 piece of paper for the formulas.
2. Throughout this paper, $N(\mu, \sigma^2)$ will denote a normal distribution with mean μ and variance σ^2.
3. This is a 1 hr 30 min exam.

Problem A. True or False: Please Explain your answers in detail. Partial credit will be given (50 points)

1. $E(X+Y) = E(X) + E(Y)$ only if the random variables X and Y are independent.

   False. By the plug-in rule, this relation holds irrespective of whether X and Y are independent.

2. If $P(A|B) = 0.6$ and $P(B) = 0.2$ then $P(A \cap B) = 0.12$.

   True. $P(A \cap B) = P(A|B) \cdot P(B)$

3. If X is a Bernoulli random variable with probability of success, p, then its variance is $V(X) = p(1-p)$.

   True. The Bernoulli random variable is the building block of the binomial ($n = 1$) and hence has variance $p(1-p)$

4. Suppose that the amount of money spent at Disney World is normally distributed with a mean of \$60 and a standard deviation of \$15. Then approximately 45% of people spend more than \$70 per visit.

False. $z_{70} = \frac{70-60}{15} = 0.67$ *and* $1\text{-}F(z_{70}) = 0.25$*. Hence, about 25% of people spend more than \$70 per visit.*

5. Suppose that X is Binomially distributed with $E(X) = 5$ and $Var(X) = 2$, then $n = 10$ and $p = 0.5$.

False. The variance of a $Bin(10, 0.5)$ *random variable is* $np(1-p) = 2.5$ *and not* 2.

6. The Red Sox are to play the Yankees in a seven game series. Assume that the Red Sox have a 50% chance of winning each game, with the results being independent of each other. Then the probability of the series ending 4-3 in favor of the Red Sox is $0.5^7 = 0.0078$.

False. The total wins of the Red Sox is a binomial random variable with $n = 7$ *and* $p = 0.5$*. The probability of* 4 *wins is then* $\binom{7}{4} 0.5^4 0.5^{7-4} = 35 \cdot (0.5)^7 = 0.27$*. Alternatively, if the series is stopped as soon as one time wins 4 matches, the probability of a 4-3 outcome in favor of the Red Sox is* $\binom{6}{3} 0.5^3 0.5^{6-3} \cdot 0.5 = 0.16$ *since the teams should first reach a 3-3 tie.*

7. Suppose that for a certain Caribbean island the probability of a hurricane is 0.25, the probability of a tornado is 0.44 and the probability of both occurring is 0.22. Then the probability of a hurricane or a tornado occurring is 0.05.

False. $P(H \cup T) = P(H) + P(T) - P(H \cap T) = 0.25 + 0.44 - 0.22 = 0.47$

8. A firm believes it has a 50-50 chance of winning a $80,000 contract if it spends $5,000 on a proposal. If the firm spends twice this amount, it feels its chances of winning improve to 60%. If the firm wants to maiximsie its expected value then it should spend $10,000 to try and gain the contract.

True. E(spend $5k) = 0.5·80000 - 5000 = 35000

E(spend $10k) = 0.6·80000 - 10000 = 38000

9. A Chicago radio station believes 30% of its listeners are younger than 30. Out of a sample of 500, the probability that more than 200 are under 30 is 0.25.

False. The number of listeners under 30 is a binomial variable with $n = 500$ *and* $p = 0.3$. *This can be approximated by a* $N(np, np(1-p))$. *The Z-value associated with 200 is* $Z = \frac{200-150}{\sqrt{105}} = 4.88$ *and* $1 - F(z) = 0$. *So there is a zero probability of finding more than* 200 *listeners under* 30.

10. A wine producer claims that the proportion of customers who cannot distinguish his product from grape juice is at most 5%. For a sample of 100 people he finds that 10 fail the taste test. He should reject his null hypothesis $H_0 : p = 0.05$ at the 5% level.

True. $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.1 - 0.05}{\sqrt{\frac{0.05 \cdot 0.95}{100}}} = 2.29$

$z_{\alpha/2} = 1.96$

Reject the null-hypothesis since $|z| > z_{\alpha/2}$

Problem B. (20 points)
Assume that the probability is 0.95 that a jury selected to try a criminal case will arrive at the correct verdict whether innocent or guilty. Further, suppose that the 80% of people brought to trial are in fact guilty.

1. Given that the jury finds a defendant innocent what's the probability that they are in fact innocent?
2. Given that the jury finds a defendant guilty what's the probability that they are in fact guilty?
3. Do these probabilities sum to one?

Let I denote the event of innocence and let VI denote the event that the jury proclaims an innocent verdict.

1. We want $P(I|VI)$ which is given by Bayes' rule

$$P(I|VI) = \frac{P(VI|I)P(I)}{P(VI)}$$

   where, by the law of total probability $P(VI) = P(VI|I)P(I) + P(VI|\bar{I})P(\bar{I})$ Hence $P(VI) = 0.95 \cdot 0.2 + 0.05 \cdot 0.8 = 0.23$
   Hence $P(I|VI) = \frac{0.95 \cdot 0.2}{0.23} = 0.83$

2. Similarly

$$P(G|VG) = \frac{P(VG|G)P(G)}{P(VG)} = \frac{P(VG|G)P(G)}{1 - P(VI)} = \frac{0.95 \cdot 0.8}{1 - 0.23} = 0.99$$

3. No. $P(I|VI) + P(G|VG) \neq 1$ because they are conditioned on different events.

Problem C. (20 points)
In a pre-election survey of 435 voters, 10 indicated that they planned to vote for Ralph Nader.
In a separate survey of 500 people, 250 said they planned to vote for George Bush.

1. Perform a hypothesis test at the 5% level for the hypothesis that Nader will get 3% or less of the vote.

2. Find a 95% confidence interval for the difference in the proportion of people that will vote for George Bush versus Ralph Nader.

1. $H_0 : p \leq 0.03$

   $H_1 : p > 0.03$

   $z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ where $\hat{p} = \frac{10}{435} = 0.023$ and $p_0 = 0.03$

   Hence, $z = \frac{0.023-0.03}{\sqrt{\frac{0.03 \cdot 0.97}{435}}} = -0.86$

   $z_\alpha = 1.65$ (one-sided test!) so we do not reject H_0 as $z < z_\alpha$

2. We know that the confidence interval is given by $(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ and we have $\hat{p}_1 = \frac{250}{500} = 0.5$ and $\hat{p}_2 = \frac{10}{435} = 0.023$. Plugging everything into the formula we find:

   $(0.5 - 0.023) \pm 1.96\sqrt{\frac{0.5 \cdot 0.5}{500} + \frac{0.023 \cdot 0.977}{435}} = [0.431, 0.523]$

Problem D. (20 points)
The following probability table describes the daily sales volume, X, in thousands of dollars for a salesperson for the number of years Y of sales experience for a particular company.

	Y			
X	1	2	3	4
10	0.14	0.03	0.03	0
20	0.05	0.10	0.12	0.07
30	0.10	0.06	0.25	0.05

1. Verify that this is a legal probability table

   All probabilities are between zero and one, and the entire table adds to one

2. What is the probability of at least two years experience?

   $P(Y \geq 2) = 0.19 + 0.40 + 0.12 = 0.71$

3. Calculate the mean daily sales volume

   $E(X) = 10 \times 0.20 + 20 \times 0.34 + 30 \times 0.46 = 22.6$, *that is $22,600*

4. Given a salesperson has three years experience, calculate the mean daily sales volume.

   First, the conditional probability distribution $p(X|Y)$ *is given by,* $0.03/0.4, 0.12/0.4, 0.25/0.4$.

   Hence $E(X|Y = 3) = 10 \times 0.075 + 20 \times 0.3 + 30 \times 0.625 = 25.5$, *that is $25,500*

5. A salesperson is paid $1000 per week plus 2% of total sales. What is the expected compensation for a salesperson?

   Assuming a 5-day workweek, $E(Comp) = 1000 + 0.02E(X_1 + X_2 + X_3 + X_4 + X_5)$

   $= 1000 + 0.02 \cdot 5 \cdot E(X) = 1000 + 0.02 \cdot 5 \cdot 22600 = \$3,260$

   With a 7-day week, $E(Comp) = 1000 + 0.02 \cdot 7 \cdot 22600 = \$4,164$